CALCULATION OF CONVERGENT SHOCK WAVES

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Shock waves (SW) which converge to their axis or center of symmetry are a special object of investigation in the problem of obtaining a dense, high-temperature plasma for power plants and in shock tubes which operate on the principle of the accumulation of energy stored in an electrical or chemical source. The subject is also important in other scientific problems. Since similarity solutions for convergent SW's [1-5] are applicable only within a certain region of the front, the main method of theoretically studying the problem has been to combine qualitative physical analysis with numerical calculation in a complete formulation which accounts for the most important physical processes [6].

In a similarity solution – representing the asymptote in the neighborhood of the axis or the center of symmetry – the temperature, pressure, and velocity increase without limit. Here, as indicated in [3], dissipative effects (viscosity, heat conduction) do not always lead to a limitation on the concentration of energy density. At the same time, finite values of these quantities are always obtained in numerical calculations performed by finite-difference methods. This follows, for example, from [6-11]. The authors of these studies numerically modeled several problems on convergent SW's. In particular, the authors of [6] stated the necessary conditions of applicability of the finite-difference approach for such problems. These conditions require that the numerical solution coincide with the similarity solution at moments of time before and after focusing and that it subsequently merge with the asymptote for a point explosion. Here, we use the example of convergent SW's in an ideal gas with the adiabatic exponent $\gamma = 7/5$ and 5/3 to show that numerical solutions found by the Lagrangian methods with artificial viscosity converge to the similarity solution [12]. Comparison of results (each of which satisfies the conditions in [6]) obtained on different meshes with a monotonic reduction in the size of the calculated region of the center of symmetry makes it possible to evaluate the minimum size of this region sufficient to achieve the prescribed accuracy in calculating the flow field corresponding to convergence and reflection of the SW's. We also present results of calculations of the focusing of SW's in metal spheres, which can be described by the model of an ideal elastoplastic isotropic medium [13] and Tillotson's equation of state [14]. Here, we examine the parameter changes in the substance in the focusing region for different materials (Al, Fe, W) and shock-wave intensities.

1. Calculations of convergent SW's in a gaseous medium are usually confined to examination of a certain neighborhood of the center or axis of symmetry. The size of this region, on the order of the thickness of the from, is determined by dissipative processes (viscosity, heat conduction, radiation) [6-11]. However, the SW front in an ideal gas has no thickness, and the Landau–Stanyukovich–Guderley similarity solution [1, 2] predicts an unlimited accumulation of energy density, i.e. an infinite increase in temperature, pressure, and mass velocity $(T \sim p \sim R^{-2(k-1)},\ u \sim R^{-(k-1)})$ on the front of SW's converging to the center of symmetry (R = 0). In connection with this, it becomes necessary to consider the behavior of the numerical solution and its agreement with the similarity solution as the SW'S come closer to converging at and being reflected from the center of symmetry.

Let a spherical layer of heated gas with an initial pressure $p/p_0 = 480$ and a density $\rho/\rho_0 = 8$ begin at $t > 0$ to expand into a stationary medium with the parameters $p_0 = 27$, $\rho_0 = 2.7$. The stationary medium is filling a cavity of the radius $R_0 = 30$. Here, the gasdynamic quantities are presented in dimensionless units. Using the relations $u_* = R_*/t_*$, $p_*/\rho_* = u_*^2$ we can obtain the true values of the sought quantities by selecting the characteristic dimensions of length and time.

The decay of the discontinuity and the convergence of the resulting SW toward the center of symmetry were calculated on an irregular mass grid. The grid was constructed from the central cell, with a linear dimension ΔR_1, and mass $\rho_0(\Delta R_1)^3$, to the boundary of the cavity R_0 with the progression 1.1. For the two variants of the problem, with $\gamma = 7/5$ (a) and 5/3

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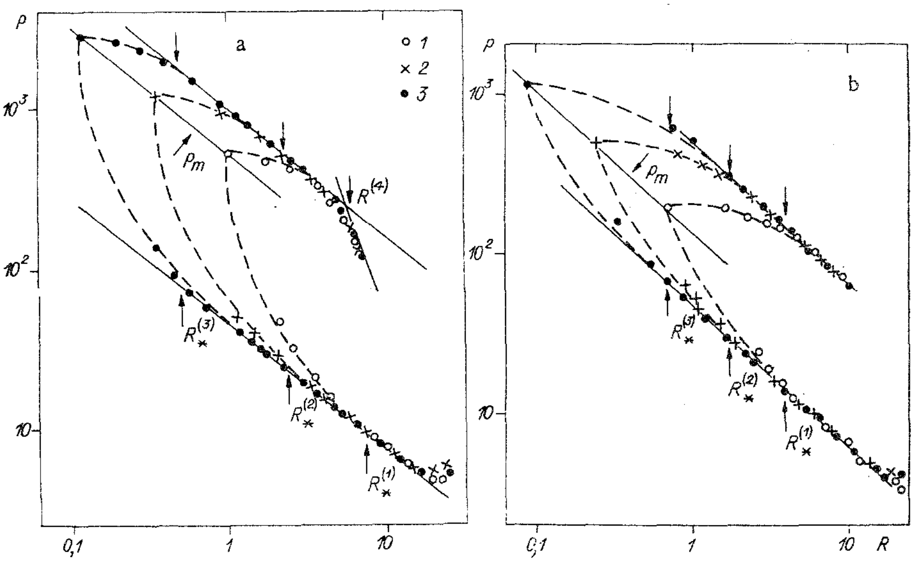

Fig. 1

(b), Fig. 1 shows the dependence of the pressure on the front on the distance to the center for the convergent (lower points) and reflected (upper points) SW's for the similarity solution k = 1.395 (a) and 1.452 (b) (solid lines) and the numerical solution with different values of the initial dimension $\Delta R_1^{(1-3)} = 3,\ 1,\ 0.3$ and different numbers of cells on the interval $0 \le R \le R_0$ 50, 70, and 100 (points 1-3). The dashed lines show the non-self-similar change in pressure in the convergent and reflected flows. The number of cells in the heated region was constant for all variants (50). It can be seen from Fig. 1 that satisfactory agreement of the numerical and similarity solutions begins with distances $R = 0.5R_0$ and is maintained to a certain neighborhood of the center $R \approx R_*^{(1-3)}$, dependent on the mesh of the computational grid. At the moment of focusing $t = t_c$, the maximum pressure in the central cell is a finite quantity for each variant. It increases as the solution approaches the center in accordance with the law $p_m \sim (\Delta R_1)^{-2(k-1)}$, where ΔR_1 is the current size of the compressed first cell at $t = t_c$. Beginning with a certain distance $R^{(4)}$ (Fig. 1a), the front of the reflected SW merges with the asymptote associated with a Sedov–Taylor strong explosion.

Figure 2 shows the calculated density profiles over 50 cells during convergence and reflection of SW's at the moments of time $(a - \gamma = 7/5)$ $t = 14;\ 16;\ 17{,}5;\ 18{,}4;\ 19{,}2;\ 20;\ 20{,}7;\ 22{,}5;\ 25;\ 27$ μsec, $(b - \gamma = 5/3)$ $t = 15;\ 16;\ 17;\ 17{,}6;\ 18;\ 18{,}6;\ 19{,}1;\ 20;\ 22;\ 24$ μsec – lines 1-10, respectively. The density of the gas, equal on the SW front to $\rho_f = \rho_0(\gamma + 1)/(\gamma - 1)$ at the moment of collapse $t = t_c$ reaches the limiting value $\rho_\ell = 7.34\ \rho_0\ (\gamma = 5/3)$ and $21.7\ \rho_0\ (\gamma = 7/5)$, in the central cell, while after SW reflection it reaches the maximum value $\rho_m = 31{,}3\ \rho_0\ (\gamma = 5/3)$ and $134.9\ \rho_0\ (\gamma = 7.5)$. The calculated values of p_ℓ and ρ_m for $\gamma = 7/5$ are closer to the values reported in [4] than in [5].

The results indicate satisfactory agreement between the numerical solution and the similarity solution. The conditions in [6] are satisfied for all of the computational grids, with 50, 70, and 100 cells. The SW converges to the center of symmetry. The conditions for the divergent flow after reflection are also satisfied. However, these results do not allow us to properly choose among the theoretical variants considered, the latter differing both in the distribution of the gasdynamic quantities over the radius and in the computing time.

Let us examine the theoretical dependences of the total energy $E = \rho u^2/2 + p(\gamma - 1)$ on the mass of the gas for $\gamma = 7/5$ (a) and $\gamma = 5/3$ (b) that were obtained on grids of 50, 70, and 100 cells (points 1-3) at the moment of time $t \approx t_c$ (Fig. 3). It is evident that the theoretical curves converge to a certain limiting profile. In the present case, this profile nearly coincides with that obtained on grids of 100 and 130 cells and with the similarity curve depicting the well-known energy relation in the similarity region $R_f \le R < R_a$ behind the front $E \sim R^{5-2k}$ [4, 5]. Similar convergence is seen for the pressure, density, and velocity curves constructed as a function of the mass of the gas ρR^3 or the coordinate R.

Detailed analysis of the agreement between the numerical solutions and the similarity solution at $t < t_c$, $t \ge t_c$ suggests that there is a region of the center (or axis) of symmetry

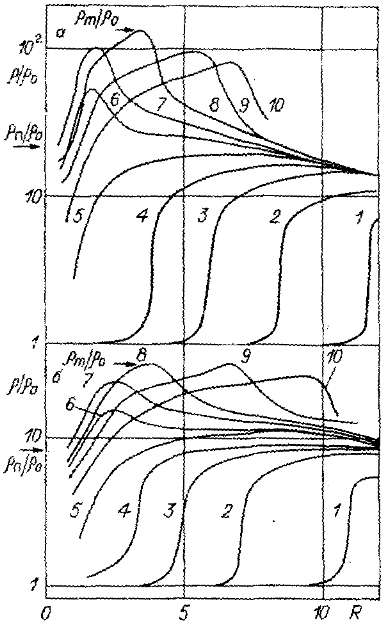

Fig. 2

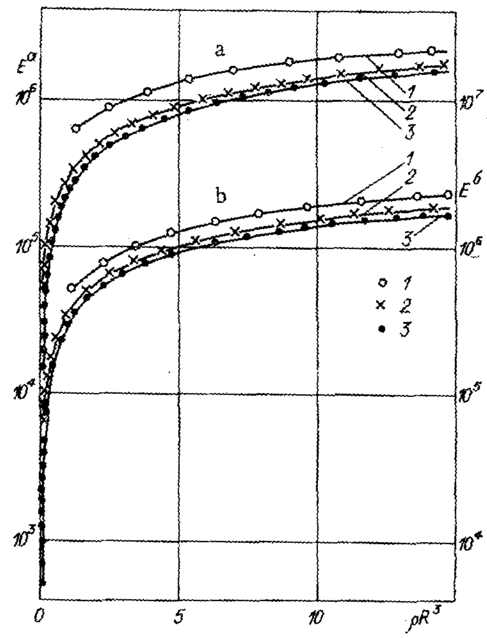

Fig. 3

R_* such that, for any initial conditions $R_0 \gg R_*$, the solution of the problem of the convergence and divergence of SW's in the region $R > R_*$ does not change. Despite the fact that the energy density here increases without limit, the total energy in the region $R < R_*$ is so small that, within a certain specified range of accuracy, the flow over the entire remainder of the region is independent of it. The value of $R_* = n\Delta R_1^*$ – where ΔR_1^* is the size the central cell at $t = t_c$ and n is the number of cells over which the SW front is approximated by a specific difference scheme – can be determined by calculation if the accuracy of the convergence for the gasdynamic quantities is specified.

Thus, for an ideal gas, the numerical approach not only makes it possible to correctly calculate the convergence of an SW to a region of the center of symmetry which is as small as desired, it also evidently permits determination of a certain finite size of this region for which the accuracy of the calculations are not improved. In particular, for the variants examined here, the results obtained with $\Delta R_1^{(4)} = 0.1$ and 130 cells nearly coincide everywhere – except for the focusing region $R < R_*$ – with the data obtained with $\Delta R_1^{(3)} = 0.3$ and 100 cells. Then $\Delta R_1^* \leqslant \Delta R_1^{(3)}$.

2. The study of convergent SW's in condensed media is of analytical interest, particularly in connection with the search for information on the thermodynamic properties of substances at high pressures and temperatures [15, 16]. The complexity of mathematically describing the focusing of an SW in a nonideal medium allows investigators to obtain a similarity solution only in isolated cases, such as with the Grüneisen equation of state [16]. A more realistic formulation of the problem requires the use of numerical methods.

In accordance with [13], the system of equations in Lagrangian coordinates for the case of spherical symmetry (the point above the quantities denoting a time derivative along the path of a particle of the medium) is written in the form

$$\dot{u} = v_0 \left(\frac{R}{M}\right)^2 \frac{\partial \Sigma_R}{\partial M} + 2\frac{\Sigma_R - \Sigma_\theta}{R}, \quad \dot{R} = u,$$

$$v = v_0 \left(\frac{R}{M}\right)^2 \frac{\partial R}{\partial M}, \quad \dot{\varepsilon} - v(s_1\dot{e}_1 + 2s_2\dot{e}_2) + (p+q)\dot{v} = 0,$$

$$\Sigma_R = -(p+q) + s_1, \quad \Sigma_\theta = -(p+q) + s_2.$$

Here, Σ_R and Σ_θ are the radial and tangential stresses; M is the mass coordinate; ϵ is the specific internal energy; $v = 1/\rho$ is the specific volume; q is pseudoviscosity; e_1, e_2, and e_3 are components of the strain vector; s_1, s_2, s_3 are components of the stress deviator.

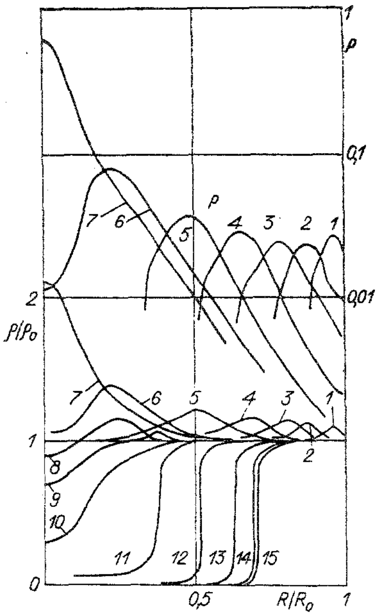

Fig. 4

The following relations were used to calculate the elastoplastic properties:

$$\dot{s}_1 = 2\mu\left(\dot{e}_1 - \frac{\dot{v}}{3v}\right),\ \dot{s}_2 = 2\mu\left(\dot{e}_2 - \frac{\dot{v}}{3v}\right),\ \dot{s}_3 = 2\mu\left(\dot{e}_3 - \frac{\dot{v}}{3v}\right),$$

$$\dot{e}_1 = \frac{\partial u}{\partial R},\ \dot{e}_2 = \frac{u}{R},\ \dot{e}_3 = \dot{e}_2.$$

The components of the stress deviator s in the region of plastic flow of the medium were corrected under the von Mises condition $s_1^2 + s_2^2 + s_3^2 \leqslant \frac{2}{3}Y_0^2$, where the yield point Y_0 and the shear modulus μ were assumed to be equal to: Y_0 = 0.3, 0.6, 3 GPa, μ = 24.8, 80, 154 GPa for Al, Fe, and W, respectively.

Let us examine the change in the parameters in the focusing region for SW's of different intensities leaving a layer with the thickness ΔR and the total energy E_0 on the surface of spheres of different materials (Al, Fe, W) with the radius R_0 = 0.2125 cm.

At $\Delta R/R_0 \equiv \Delta_g \leq 0.1$ (E_0 = 0.11 kJ) for Al, $\Delta_g \leq 0.2$ (0.25 kJ) for Fe, and $\Delta_g \leq 0.5$ (0.76 kJ) for W, a convergent SW in the metal remains weak due to loss of energy during completion of work on elastoplastic deformation of the sphere. This is expressed in the finiteness of the increase in energy density and other gasdynamic quantities at the center of symmetry. For example, for Fe at ρ_0 = 7.83 g/cm^3, the maximum energy density and maximum specific internal energy converge to constant values ρ_m/ρ_0 = 1.50, 1.55, 1.56; ϵ_m = 3.49, 4.62, 4.69 kJ/g with an increase in the number of grid cells to 30, 50, and 70, respectively.

At $\Delta > \Delta_g$ for all of the test materials, focusing of an SW leads to an increase in energy density in the neighborhood of the center to values greater than the energy of vaporization of the substance. Focusing also leads to entrainment of mass from the center after reflection of the SW, i.e. to the formation of a cavity of a certain size. The authors of [17] experimentally observed and numerically modeled the formation of a closed cavity at the centers of metal spheres symmetrically exposed to an electron beam. In our case, as calculations showed, there are narrow ranges of loading $\Delta \equiv \Delta_\text{п} \approx 0.1-0.2$ (Al), 0.2–0.3 (Fe), 0.5 – 0.7 (W), for which a cavity of finite dimensions is also formed inside the sphere. The dynamics of its formation in iron at Δ = 0.3 is evident from Fig. 4, which shows profiles of pressure p (TPa) and relative density ρ/ρ_0 over the radius at the moments of time t = 0.13; 0.16; 0.2; 0.25; 0.32; 0.4; 0.44; 0.5; 0.53; 0.63; 1; 1.59; 2.51; 3.98; 6.3 μsec – lines 1-15, respectively. It can be seen that growth of the cavity ceases at t ≈ 6.3 μsec.

With an increase in Δ and E_0, the role of energy dissipation in elastoplastic flow begins to decrease and the SW focusing process changes to a regime similar to compression in

TABLE 1

Δ	E_0, kJ	p_m, TPa		
		number of cells		
		30	50	70
0,1	0,11	0,163	0,209	0,217
0,31	0,43	0,449	0,875	1,128
1,0	2,25	1,47	5,89	29,4
3,0	20,2	4,09	18,8	89,2
10	426	22,3	96,6	851,2

a gaseous medium with an infinite increase in energy density at the center. This may include a change in the ratio of energy expended on elastoplastic deformation of an iron sphere to the work of pressure in the SW $v(s_1de_1 + 2s_2de_2)/pdv \sim 10;\ 1;\ 0.1;\ 0.01$ at Δ = 0.1, 0.3, 1, 10, as well as a divergence of the values of maximum pressure p_m at the center of the sphere. Table 1 shows values of p_m as a function of the intensity of the external effects Δ and E_0.

The calculated results examined above demonstrate the reliability of numerical solutions of the problem of convergent SW's, as is confirmed by the agreement with the similarity solution (gaseous medium) and the agreement between theoretical and experimental data (effect of formation of a closed cavity in an elastoplastic medium). As regards the problem of the accumulation of energy density in a convergent SM, it can be concluded either that an infinite increase in the quantities occurs in a certain neighborhood of the focal point and has no effect on motion outside it or is limited by the corresponding actual physical mechanism of dissipation (as was indicated in [18]) if this mechanism plays a significant role in the energy balance of the cumulative flow.

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STABILITY OF SYMMETRICAL COMPRESSION OF A CYLINDRICAL LINER MODELING A SYSTEM OF WIRES

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Low-inductance multiwire devices ("arrays") [1], used as the load in pin-diodes, have made it possible to obtain plasmas characterized by high velocities (~10^7 cm/sec) and extreme parameters. This had in turn made such plasmas a promising tool in studies of powerful sources of electromagnetic radiation and dense-plasma generators [2]. The multiwire devices are also of interest for modeling the dynamics of the compression of cylindrical liners and z-pinches. The study of instabilities which disturb the synchronicity of the convergence of conductors at the center of the system is an important goal [3]. Here, we analyze the stability of the symmetrical collapse of an "array" with allowance for the mutual inductive effect of the currents and the finite ohmic resistance of the conductors. By using an asymptotic solution, the results are extended to the case of a solid liner.

<u>Formulation of the Problem.</u> We will examine a system of N rectilinear conductors (wires) with current. The conductors are positioned between two plane electrodes and close a circuit with the voltage source E, external inductance L_{ext}, and external resistance Ω_{ext}. It is assumed that the wires remain parallel to the z axis during motion and have transverse dimensions much smaller than the characteristic spacing. In this case, the motion of the liner reduces to the motion of point masses in the plane (x, y). We will use a Lagrangian formulation of the problem [4] to obtain the corresponding equations of motion with allowance for the changing inductance of the system. Each conductor is described by three generalized coordinates. Two of these coordinates (x, y) describe the position of the conductor, while the third coordinate Q gives the magnitude of the transmitted charge and corresponds to an "electrical" degree of freedom. The Lagrangian of the system, comprised of the kinetic energy of the wires, the energy of the magnetic field, and the energy of the external source, has the form

$$\mathscr{L} = \frac{1}{2}\sum^{\alpha} M_\alpha \dot{X}_\alpha^2 + \frac{1}{2}\sum^{\alpha\beta} L_{\alpha\beta}\dot{Q}_\alpha\dot{Q}_\beta + \frac{1}{2}L_{\text{ext}}\left(\sum^{\alpha}\dot{Q}_\alpha\right)^2 + E\sum^{\alpha} Q_\alpha; \quad (1)$$

$$L_{\alpha\beta} = 2lc^{-2}\ln\left(R_\infty/|X_\alpha - X_\beta|\right),\ \alpha \neq \beta, \quad L_{\alpha\alpha} = 2lc^{-2}\ln\left(R_\infty/r_\alpha\right). \quad (2)$$

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